Interrogation du 14/02

Ex. 1 $\alpha_{n+2}+5\alpha_{n+1}+6\alpha_n=n$

Equation homogène :

(*) $\alpha_{n+2}+5\alpha_{n+1}+6\alpha_n=0$

Equation caractéristique :

$$\lambda^2+5\lambda+6=0 \quad \longrightarrow \quad \lambda_1=-2,\ \lambda_2=-3$$

La solution générale de l'équation homogène (*) :

$$\alpha_n = C(-2)^n + D(-3)^n$$

Solution particulière :

$$\alpha_n=\beta n+\gamma \implies \begin{cases} \alpha_{n+1}=\beta(n+1)+\gamma \\ \alpha_{n+2}=\beta(n+2)+\gamma \end{cases}$$

$$\alpha_{n+2}+5\alpha_{n+1}+6\alpha_n = \beta(n+2)+\gamma+5\beta(n+1)+5\gamma+6\beta n+6\gamma =$$

$$= 12\beta n+(12\gamma+7\beta)=n$$

d'où

$$\left.\begin{array}{r} 12\beta = 1 \\ 7\beta+12\gamma=0 \end{array}\right\} \implies \begin{array}{l} \beta=\frac{1}{12} \\ \gamma=-\frac{7}{144} \end{array}$$

$$\alpha_n=\frac{n}{12}-\frac{7}{144}$$

La solution générale de l'équation initiale :

$$\alpha_n = C(-2)^n+D(-3)^n+\frac{n}{12}-\frac{7}{144}.$$

Ex. 2 $I=\displaystyle\int\frac{x\,dx}{(x^2-1)(x-2)(x+3)}$

$$\frac{x}{(x-1)(x+1)(x-2)(x+3)} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-2}+\frac{D}{x+3}$$

$$A=\left[\frac{x}{(x+1)(x+3)(x-2)}\right]_{x=1} = \frac{1}{2\cdot4\cdot(-1)} = -\frac{1}{8}$$

$$B=\left[\frac{x}{(x-1)(x-2)(x+3)}\right]_{x=-1} = \frac{-1}{-2(-3)\cdot2} = -\frac{1}{12}$$

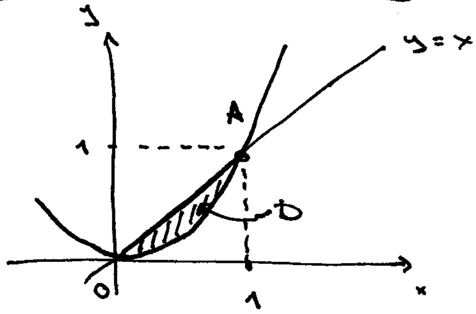
$$C = \left[\frac{x}{(x^2-1)(x+3)}\right]_{x=2} = \frac{2}{3\cdot 5} = \frac{2}{15}$$

$$D = \left[\frac{x}{(x^2-1)(x-2)}\right]_{x=-3} = \frac{-3}{8\cdot(-5)} = \frac{3}{40}$$

Donc

$$I = -\frac{1}{8}\ln(x-1) - \frac{1}{12}\ln(x+1) + \frac{2}{15}\ln(x-2) + \frac{3}{40}\ln(x+3) + \text{const}$$

Ex. 3) Domaine D :



Point d'intersection A : $(1,1)$

$$x = x^2 \Rightarrow x = 0,\ \boxed{x=1}$$

$$\iint_D (x+y)\,dx\,dy = \int_0^1 \left(\int_{x^2}^{x} (x+y)\,dy \right) dx = \int_0^1 \left[xy + \frac{y^2}{2} \right]_{y=x^2}^{y=x} dx =$$

$$= \int_0^1 \left\{ x(x-x^2) + \frac{x^2-x^4}{2} \right\} dx =$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} + \frac{x^3}{6} - \frac{x^5}{10} \right]_{x=0}^{x=1} = \frac{1}{3} - \frac{1}{4} - \frac{1}{10} + \frac{1}{6} = \frac{3}{20}$$